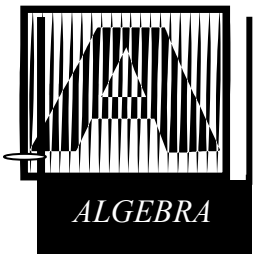
MAKE THE GRADE



QUESTIONS 1 - 9

ANSWERS ❶ = 1 mark

1. Find the equation of the line passing through (6,5) and perpendicular to the line y = 3x + 4.

Gradient of perpendicular line = $-\frac{1}{3}$ ❶

$y = -\frac{1}{3}x + c$ ❶

The line passes through (6,5) so:

$5 = (-\frac{1}{3} \times 6) + c$ ❶

$5 = -2 + c$

$c = 7$

$y = -\frac{1}{3}x + 7$ ❶

(4)

2. Rearrange the following expressions in descending order if:

a^{-2} $a^{\frac{1}{2}}$ a^{0} $a^{-\frac{1}{2}}$

i) a = 16

$16^{-2} = \frac{1}{256}$ ❶ $16^{\frac{1}{2}} = 4$ ❶ $16^{0} = 1$ ❶ $16^{-\frac{1}{2}} = \frac{1}{4}$ ❶

Descending order:

$a^{\frac{1}{2}}$ a^{0} $a^{-\frac{1}{2}}$ a^{-2} ❶ **correct order**

(5)

ii) a = $\frac{1}{81}$

$(\frac{1}{81})^{-2} = (\frac{81}{1})^{2} = 81^{2}$ ❶ $(\frac{1}{81})^{\frac{1}{2}} = \frac{1}{9}$ ❶ $(\frac{1}{81})^{0} = 1$ ❶ $(\frac{1}{81})^{-\frac{1}{2}} = (\frac{81}{1})^{\frac{1}{2}} = 9$ ❶

Descending order:

a^{-2} $a^{-\frac{1}{2}}$ a^{0} $a^{\frac{1}{2}}$ ❶ **correct order**

3. Make t the subject of the formula in each case:

i) $P = \frac{4}{t}$

$t = \frac{4}{P}$ ❶

(1)

ii) $W = \frac{2+3t}{t}$

$tW = 2 + 3t$ ❶

$tW - 3t = 2$

❶ $t(W - 3) = 2$

$t = \frac{2}{W - 3}$ ❶

(3)

iii) $F = \frac{s - t}{t}$

$tF = s - t$ ❶

$tF + t = s$

❶ $t(F + 1) = s$

$t = \frac{s}{F + 1}$ ❶

(3)

4. The volume of a splogoid is directly proportional to the cube of the diameter
When the volume is 324cm³, the diameter is 6cm.

i) Find a formula for the volume of a splogoid, V, in terms of the diameter, d.

$V \alpha\ d^3$

$V = kd^3$ ❶

$324 = k6^3 = 216k$

$k = \frac{324}{216} = \frac{3}{2} \qquad \Rightarrow \qquad V = \frac{3d^3}{2}$ ❶

(2)

ii) Find the diameter of a splogoid with volume 12cm³

❶ $12 = \frac{3d^3}{2} \quad \Rightarrow \quad 24 = 3d^3 \Rightarrow d^3 = 8 \Rightarrow d = 2$ ❶

(2)

5. w is inversely proportional to the positive square root of t.
When w = 5, k = 100

i) Find a formula linking w and k

$$w \propto \frac{1}{\sqrt{t}}$$

$$w = \frac{k}{\sqrt{t}} \quad \text{(1)}$$

$$5 = \frac{k}{\sqrt{100}} = \frac{k}{10}$$

$$k = 5 \times 10 = 50 \quad \Rightarrow \quad w = \frac{50}{\sqrt{t}} \quad \text{(1)}$$

(2)

ii) Using your formula find:

a) w when t = $\frac{1}{4}$

$$w = \frac{50}{\sqrt{\frac{1}{4}}} = \frac{50}{\frac{1}{2}} = 100 \quad \text{(1)}$$

(1)

b) k when w = 10

$$10 = \frac{50}{\sqrt{t}} \quad \text{(1)}$$

$$\sqrt{t} = \frac{50}{10} = 5$$

$$t = 5^2 = 25 \quad \text{(1)}$$

(4)

6. Prove that the sum of three consecutive numbers is always a multiple of 3.

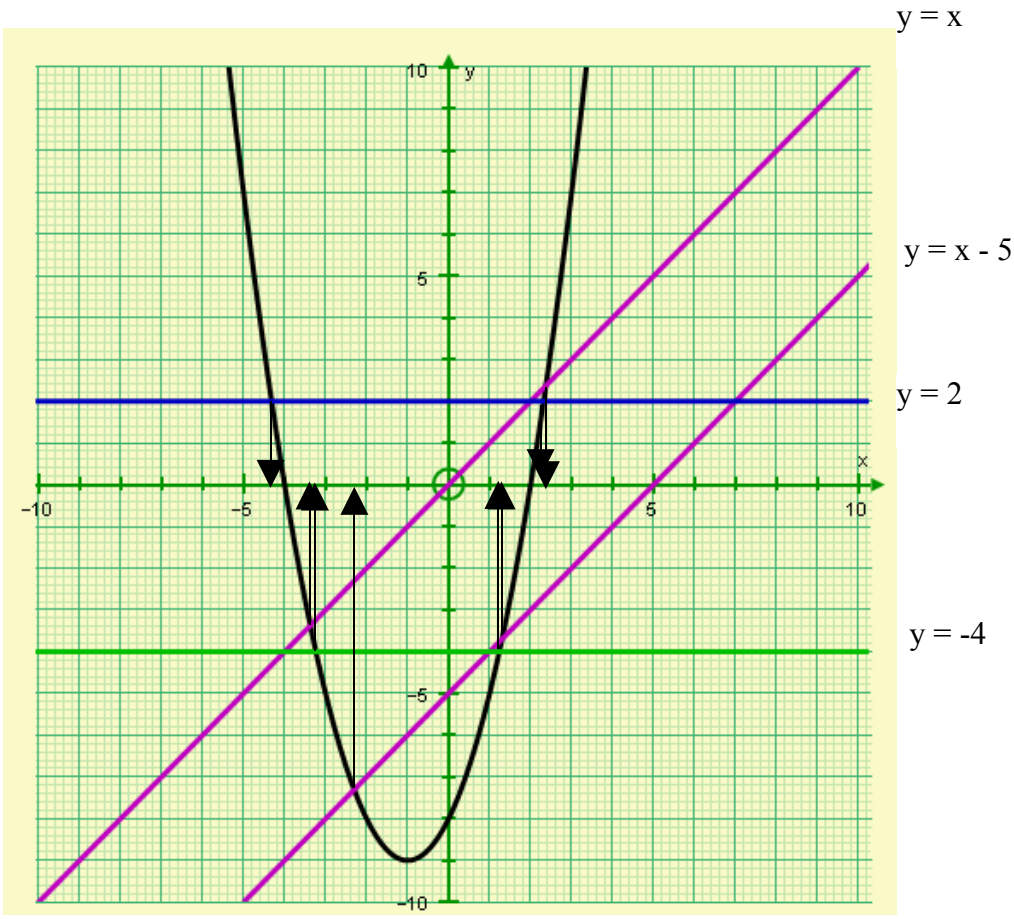
Let n, n+1 and n+2 represent the three consecutive numbers. (1)

Sum = n + n + 1 + n + 2 = 3n + 3 = 3(n + 1). (1) (1)

The sum is always a multiple of three. (1)

(4)

7. The graph of $y = x^2 + 2x - 8$ is shown below:



Use the graph to solve the following equations:

i) $x^2 + 2x - 8 = 0$

x = -4 and x = 2

ii) $x^2 + 2x - 8 = -4$ **for drawing and labelling the line y=-4**

x = -3.2 (±0.1) and x = 1.2(±0.1)

iii) $x^2 + 2x - 10 = 0$ (+2 to both sides)

$x^2 + 2x - 8 = 2$ **for correct manipulation**

for drawing and labelling the line y=2

x= -4.3 (±0.1) and x = 2.3 (±0.1)

iv) $x^2 + 2x - 8 = x$ **for drawing and labelling the line y=x**

x= -3.4 (±0.1) and x = 2.4 (±0.1)

v) $x^2 + x - 3 = 0$ (+x, -5)

$x^2 + 2x - 8 = x - 5$ **for correct manipulation of x**

for correct manipulation of constant

for drawing and labelling the line y=x-5

x= -2.3 (±0.1) and x = 1.3 (±0.1)

(17)

8. Factorise the following:

i) $x^2 - 36 = (x+6)(x-6)$ ❶ ❶

ii) $2a^2 - 50 = 2(a^2-25) = 2(a+5)(a-5)$ ❶ ❶ ❶ ❶

iii) $4s^2 - 9t^2 = (2s+3t)(2s-3t)$ ❶ ❶

iv) $2x^2 + 7x + 3 = (2x + 1)(x + 3)$ ❶ ❶ ❶ ❶

v) $3y^2 + y - 4 = (3y + 4)(y - 1)$ ❶ ❶ ❶ ❶

vi) $5x^2 - 11x + 2 = (5x - 1)(x - 2)$ ❶ ❶ ❶ ❶

(21)

9. i) Simplify $\frac{4}{x} + \frac{6}{x+2}$

$= \frac{4(x+2) + 6x}{x(x+2)} = \frac{4x + 8 + 6x}{x(x+2)} = \frac{10x + 8}{x(x+2)}$ ❶ ❶ ❶ ❶

(4)

ii) Hence, or otherwise, solve $\frac{4}{x}+\frac{6}{x+2}=2$

$\frac{10x+8}{x(x+2)}=2$ ❶

$10x+8=2x(x+2)$ ❶

$10x+8=2x^2+4x$

$0=2x^2-6x-8$ ❶

$0=(2x+2)(x-4)$

$x=-1$ ❶ or $x=4$ ❶

(5)

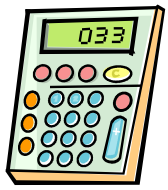
10. Solve the following, giving your answers to 1 decimal place.

i) $x^2+5x+1=0$

$x=\frac{-5\pm\sqrt{5^2-(4\times1\times1)}}{2}=\frac{-5\pm\sqrt{21}}{2}$ ❶ ❶ ❶

$x=-0.2$ ❶ or $x=-4.8$ (1dp) ❶

CALCULATORS ARE NOW ALLOWED



ii) $y^2-10y+4=0$

$y=\frac{10\pm\sqrt{(-10)^2-(4\times1\times4)}}{2}=\frac{10\pm\sqrt{100-16}}{2}$ ❶ ❶

$y=\frac{10\pm\sqrt{84}}{2}$ ❶

$y=9.6$ ❶ or $y=0.4$ (to 1dp) ❶

iii) $2x^2+13x-5=2$

$x=\frac{-13\pm\sqrt{13^2-(4\times2\times(-5))}}{4}=\frac{-13\pm\sqrt{169+40}}{4}$ ❶ ❶

$x=\frac{-13\pm\sqrt{209}}{4}$ ❶

$x=0.4$ ❶ or $x=-6.9$ (to 1 dp) ❶

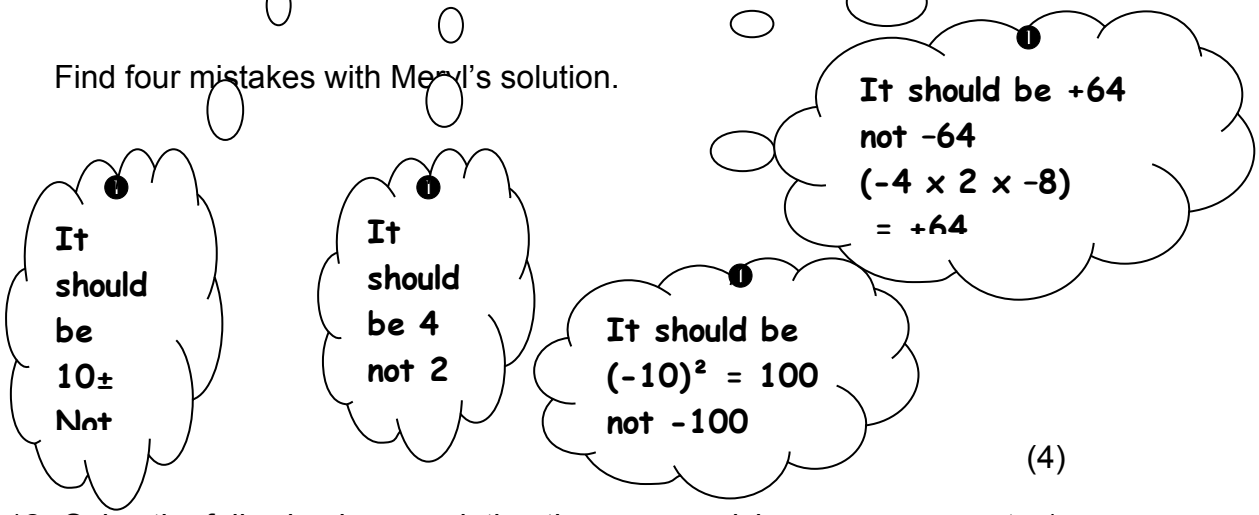
(15)

11. Meryl is solving the quadratic equation $2x^2 - 10x - 8 = 0$ using the quadratic formula.

The first part of her solution is shown below:

$$x = \frac{10 \pm \sqrt{-10^2 - 4 \times 2 \times -8}}{2} = \frac{-10 \pm \sqrt{-100 - 64}}{2}$$

Find four mistakes with Meryl's solution.



(4)

12. Solve the following by completing the square, giving your answers to 1 decimal place.

ii) $x^2 + 6x + 2 = 0$

$(x + 3)^2 - 9 + 2 = 0$

$(x + 3)^2 = 7$

$x + 3 = \pm\sqrt{7}$

$x = -3 \pm \sqrt{7}$

$x = -0.4$ or $x = -5.6$ (to 1dp)

iii) $y^2 + y - 1 = 0$

$(y + \frac{1}{2})^2 - \frac{1}{4} - 1 = 0$

$(y + \frac{1}{2})^2 = \frac{5}{4}$

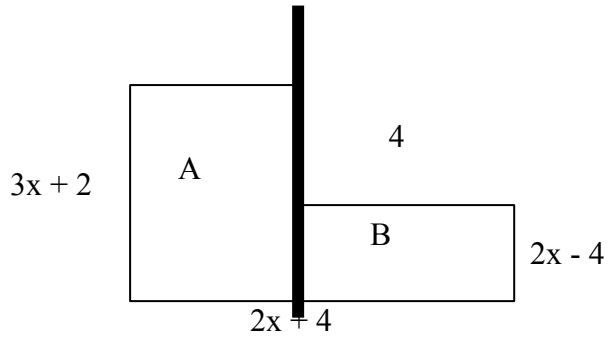
$y + \frac{1}{2} = \pm\sqrt{\frac{5}{4}}$

$y = -\frac{1}{2} \pm \sqrt{\frac{5}{4}}$

$y = 0.6$ or $y = -1.6$ (to 1dp)

(10)

13.



a) Show that the area of the shape is given by the expression $6x^2 + 12x - 16$.

Area A = $2x(3x+2) = 6x^2 + 4x$ ❶
Area B = $4(2x-4) = 8x - 16$ ❶

Total Area = $\underbrace{(6x^2 + 4x) + (8x - 16)}_{❶} = 6x^2 + 12x - 16$

(3)

b) If the area of the shape is 20cm², find the length of the longest side.

$20 = 6x^2 + 12x - 16$
$0 = 6x^2 + 12x - 36$ ❶
$0 = x^2 + 2x - 6$ quadratic = 0

$x = \dfrac{-2 \pm \sqrt{2^2 - (4x1x(-6))}}{2}$

or $x = \dfrac{-12 \pm \sqrt{12^2 - (4x6x(-36))}}{12}$

……

$x = 1.64575.....$ or $x = -3.645751311$

 correct use of the quadratic formula

$x = \dfrac{-2 \pm \sqrt{4 + 24)}}{2} = \dfrac{-2 \pm \sqrt{28}}{2}$

$x = 1.64575.....$ or $x = -3.645751311$ ❶

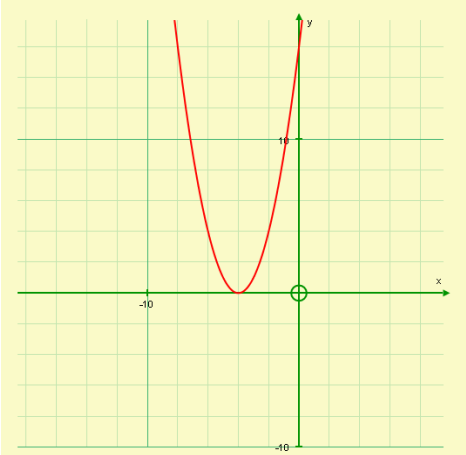
Makes no sense in this context

Longest side = 2x + 4 = (2x1.64575…) + 4 = 7.29 (to 2dp) ❶

(5)

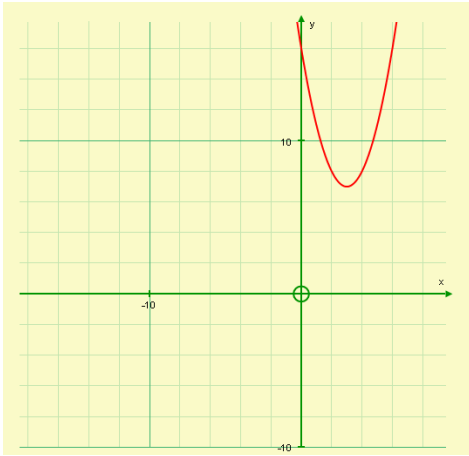
14. Match the quadratic function to the correct graph, giving reasons for your answer.

$y = x^2 + 8x + 16$ $y = x^2 - 6x + 16$



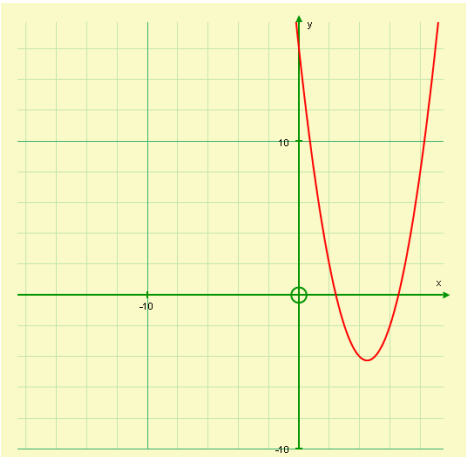
Equation: **$y = x^2 + 8x + 16$** ❶

Reason: **$x^2 + 8x + 16 = (x+4)^2$** ❶
- only one intersection with the x axis at x = -4 ❶



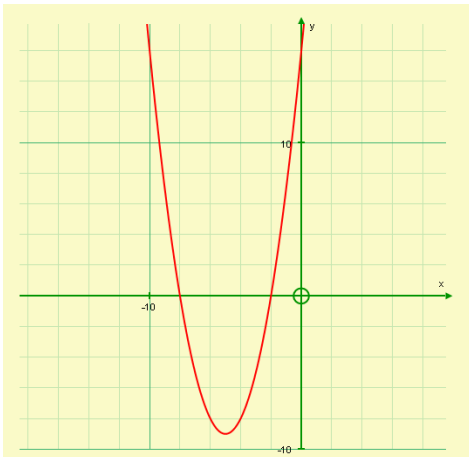
Equation: **$y = x^2 - 6x + 16$** ❶

Reason: **$x^2 - 6x + 16 = 0$ can not be** ❶ **solved using the formula (negative square root!) and so the graph does not cross the x axis** ❶



Equation: **$y = x^2 - 9x + 16$** ❶

Reason: **using the formula $x^2 - 12x + 16 = 0$ has two solutions** ❶ **$x = 2.4$ and $x = 6.6$ – two intersections with the x axis at these values.**
❶



Equation**: $y = x^2 + 10x + 16$** ❶

Reason**: $x^2 + 10x + 16 = (x+2)(x+8)$ –** **two intersections with the x axis at x** ❶ **= -2 and x = -8**
❶

(12)

Did you 'Make the Grade'?

Skill	Qu	☺	☹
I can find the equation of a perpendicular line	1		
I can evaluate algebraic expressions involving negative and fractional powers	2		
I can rearrange more complex formulae including when variables are given twice	3		
I can solve direct proportion problems	4		
I can solve inverse proportion problems	5		
I can prove simple statements	6		
I can solve quadratic equations graphically	7		
I can factorise quadratics using the difference of two square	8 i,ii,iii		
I can factorise harder quadratics (a>1)	8 iv, v, vi		
I can simplify and solve simple equations involving algebraic fractions	9		
I can solve quadratic equations using the quadratic formula	10,11,13		
I can solve simple quadratic equations using completing the square	12,(13)		
I can sketch the graphs of quadratic functions of the form $y = x^2 + bx + c$	14		

☺ *Yippee!! - I got all the questions correct.*

☹ *I made mistakes and need to practise this topic more.*

Top 3 topics I need to revise are
☺
☺
☺

If your score is:

91 - 130 - Well done. You are definitely working at grade A with your algebra skills. Bring on the A*!

45 –90 - Promising work – make sure you ask for help and revise the topics that you had difficulty with. You can still get that A !!!

0 – 45 - Serious revision and help is needed if you are going to get that A. Sort it out now. Don't wait any longer