Welcome to a 2008 UKMT TMC Regional Final

Here are some warm-up questions to get your brains working.
Discuss them with each other and with your teacher. **NO CALCULATORS!**

Question 1

Find two different ways of expressing 4104 as the sum of two cubes.

Question 2

Which numbers are increased by 500% when they are squared?

Question 3

Emily does not want to admit her age. She says ''I'm 45 years old, if you don't count Saturdays or Sundays''. What is Emily's true age?

Question 4

The difference between a three-digit number and a two-digit number is 987. How many pairs of numbers have this property?

Question 5

U	U	U	U
K	K	K	K
M	M	M	M
T	T	T	T

Can you find the 68 ways to spell out 'UKMT' through this grid? You may move across an edge or through a corner.

Question 6

	4	17
		x
5		

The diagram shows part of a magic square (in which the total for each row, column and diagonal is the same). What is the value of x?

Question 7

If $a * b$ means 'square a and subtract b', what is the value of $(-3) * (-5)$?

Question 8

The diagonals of the faces of a cuboid are, in cm, $\sqrt{45}$, $\sqrt{52}$ and 5. What is its volume?

Question 9

How many different shapes of isosceles triangles have at least one side of length 2cm and an area of 1cm^2 ?

Question 10

The annual distance travelled by Tesco's lorries is about 68 million miles, equivalent to how many round trips to the moon? (Earth to Moon $\approx 2.4 \times 10^5$ miles)

Starter Questions ~ ANSWERS

Question 1

4104 = $\mathbf{15^3 + 9^3}$ = $\mathbf{16^3 + 2^3}$

Question 2

We need to solve $x^2 = 6x$, so x = **0 or 6**

Question 3

Emily is **63** years old,
as $(45 \div 5) \times 7 = 63$.

Question 4

There are **THREE** pairs:
999 and 12,
998 and 11,
997 and 10

Question 5

U	U	U	U
2	3	3	2
5	8	8	5
13	21	21	13

= **68**

Question 6

x = **6**

QUESTION 7

$(-3)^2 - (-5) = 9 + 5 =$ **14**

Question 8

Let the dimensions be a, b, c cm.
$a^2+b^2 = 45$, $b^2+c^2 = 52$, $a^2+c^2 = 25$,
Hence $(a^2+b^2) + (b^2+c^2) - (a^2+c^2) = 72$,
giving $2b^2 = 72$, so $b = 6$, and $a = 3$, $c = 4$.
Therefore the volume is **72** cm^3.

Question 9

3 triangles. Taking 2cm as base, each has height 1cm. One triangle has just one 2cm side, two have two 2cm sides (one acute angled triangle, one obtuse angled triangle)

Question 10

About **142** round trips,
as $68\,000\,000 \div (240\,000 \times 2) = 141.666\ldots$